Cambridge International Education

Cambridge International AS & A Level

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **May/June 2024**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate’s intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Correct second term $30x$ in expansion of $(1+3x)^{10}$	B1	WWW, may be implied later.
	Correct third term $+405x^2$	B1	Ignore subsequent terms, may be implied later.
	Multiply $(2-5x)$ by *their* $30x+405x^2$ to obtain two x^2 terms only	M1	Expect $-150x^2, 810x^2$.
	Coefficient is 660	A1	Must be clearly identified. Allow final answer $660x^2$.
		4	

Question	Answer	Marks	Guidance
2(a)	State $(\frac{5}{3}\pi,\ 0)$ for point A	B1	Or exact equivalent. Allow $x=\dfrac{5}{3}\pi$ or exact equivalent.
	$x=\dfrac{19}{6}\pi$ for point B	B1	Or exact equivalent. May be implied in coordinate or vector form.
	$y=-k$ for point B	B1	May be implied in coordinate or vector form.
		3	
2(b)	Solve at least as far as $\sin^{-1}3t=k\pi$ with correct value for $\cos^{-1}\left(\frac{1}{2}\sqrt{2}\right)$	M1	Allow use of $\pi=3.14...$. Allow $\sin^{-1}3t=30$.
	$\sin^{-1}3t=\frac{1}{6}\pi$ and hence $t=\frac{1}{6}$	A1	Or exact equivalent. Can use degrees if consistent.
		2	

Question	Answer	Marks	Guidance
3(a)	State $2r + r\theta = 65$ and $\frac{1}{2}r^2\theta = 225$	**B1**	
	Form a 3-term quadratic or cubic in r or θ or $r\theta$ from correct arc and sector formula	***M1**	Condone sign errors.
	Solve *their* 3 term quadratic or cubic to obtain values of r or θ	**DM1**	Expect $2r^2 - 65r + 450 = (2r - 45)(r - 10)$ or $18\theta^2 - 97\theta + 72 = (9\theta - 8)(2\theta - 9)$.
	$r = 10$ and $\theta = 4.5$ ignore $r = 22.5$ and $\theta = \frac{8}{9}$, do not ignore $r = 0$	**A1**	**B1 SC** if no quadratic or cubic solution. If $r = 0$ included A0 or B0 SC.
		4	
3(b)	Use correct formula for area of triangle with clear use of angle being $2\pi -$ their θ	**M1**	Expect 1.783 or 102.2°, *their* θ must be reflex.
	48.9	**A1**	AWRT, WWW or a second answer. Or greater accuracy; condone absence of units.
		2	

Question	Answer	Marks	Guidance
4(a)	Use identity $\tan\theta = \frac{\sin\theta}{\cos\theta}$	**M1**	
	Use identity $\cos^2\theta = 1 - \sin^2\theta$	**M1**	
	$\pm(5\sin^2\theta + 7\sin\theta - 6 = 0)$	**A1**	
		3	

Question	Answer	Marks	Guidance
4(b)	Attempt solution of *their* 3 term equation and correct process to find at least 1 value of $\sin x$ or $\sin 2x$ or $\sin\theta$	**M1**	Expect $(5s-3)(s+2)=0,\ s=3/5$.
	$x=18.4$	**A1**	Or greater accuracy. **B1 SC** if no solution to the quadratic.
	$x=71.6$ or (90 – *their* 18.4) or greater accuracy; and no other solutions for $0° < x < 180°$	**A1FT**	WWW **B1 SC** FT if no solution to the quadratic. **B1 SC** both correct in radians, 0.322, 1.25.
		3	

Question	Answer	Marks	Guidance
5(a)	Differentiate to obtain $4x+\frac{1}{2}x^{-2}$	**B1**	OE Condone ‘+c’.
	Equate first derivative to zero and solve $4x+\dfrac{K}{x^2}=0$ as far as $x^3=k$, K and k non-zero	**M1**	Not given if ‘+c’ used.
	$x=-\frac{1}{2}$ and $y=\frac{9}{2}$	**A1**	OE **B1 SC** if no visible solution of the cubic.
		3	

Question	Answer	Marks	Guidance
5(b)	Differentiate *their* first derivative, substitute *their* x value. Substitution may be implied by a correct inequality or correct value,	**M1**	Must differentiate one term correctly. Expect $4 - x^{-3} = 12$ at $x = \frac{-1}{2}$ Alternative: substitute values of x into $\frac{dy}{dx}$. One value $x < -\frac{1}{2}$ and one value $-\frac{1}{2} < x < 0$.
	conclude minimum	**A1**	Following correct work only
		2	
5(c)	State increasing …	**B1**	
	… with clear reference to first derivative always being positive [for $x > 0$]	**B1**	Dependent on first derivative being correct. It is not sufficient to substitute values of x.
		2	

Question	Answer	Marks	Guidance
6(a)	Integrate to obtain form $k(5x-3)^{-1}$	*M1	OE
	$4(5x-3)^{-1}$	A1	Or unsimplified equivalent. Condone absence of $\ldots + c$ so far.
	Substitute $x = \frac{4}{5}$ and $y = -3$ to attempt value of c	DM1	DM0 for substituting $\left(-3, \frac{4}{5}\right)$.
	$y = 4(5x-3)^{-1} - 7$ allow f(x) *or* $f = 4(5x-3)^{-1} - 7$	A1	OE Condone $c = -7$ as the final answer providing $y = $ or $f(x) = \frac{4}{(5x-3)} + c$ OE is seen earlier. Attempts to write equation in $y = mx + c$ form scores A0. Do not ISW. Gains max 3/4.
		4	
6(b)	Carry out stretch by replacing x by $2x$ in *their* equation	M1	Award if given as the second transformation. Do not ignore sign errors.
	Carry out translation by replacing x by $x-2$ and y by $y-10$	M1	OE Award if given as the first transformation. Do not ignore sign errors.
	$y = \frac{4}{10x-23} + 3$	A1	Or similarly simplified equivalent, WWW.
		3	

Question	Answer	Marks	Guidance
7(a)	$5(3+9d)=127.5$	**B1**	OE
	$d=2.5$	**B1**	
		2	
7(b)	Attempt to find either the first term or the last term in the set by considering $1.5+2.5(n-1)>25$ or $1.5+2.5(n-1)<100$ or equivalent equations	**M1**	Using *their d*. May be implied by correct answers.
	State or imply that 11th term or 26.5 is the first in the set	**A1**	
	State or imply that 40th term or 99 is the last in the set	**A1**	
	Either use $S_{40}-S_{10}$ Or use $\frac{1}{2}n(a+l)$ with correct results for *their* d Or use $\frac{1}{2}n[2a+(n-1)d]$ with correct results for *their* d	**DM1**	*Their* 40 and 10 from correct working with *their d*. Correct values 30, 26.5 and 99 respectively. Correct values 30, 26.5 and 2.5 respectively.
	Obtain 1882.5	**A1**	OE
		5	

Question	Answer	Marks	Guidance
8	Substitute $x=0$ and attempt solution of 3-term quadratic equation in y	**M1**	If $y=0$ used can score a maximum of M0 A0 B1 M1 A0 A1FT DM1 A0, i.e. 4/8.
	-5 and 3	**A1**	**B1 SC** if no working to solve the quadratic.
	State or imply centre of circle is $(3,\ -1)$	**B1**	Condone errors which don't affect finding centre. May be implied by the correct final y coordinate.
	Attempt gradient of AC or BC	***M1**	
	$-\frac{4}{3}$ or $\frac{4}{3}$	**A1**	
	State or imply gradient of tangent is $\frac{3}{4}$ or $-\frac{3}{4}$	**A1FT**	Following *their* gradient of radius. Only FT when previous 2 marks are M1 A0.
	<u>Either</u> solve simultaneous equations (of 2 tangent equations) to find x- coordinate <u>Or</u> Substitute y-value of centre into either tangent equation	**DM1**	
	$x=-\frac{16}{3},\ y=-1$	**A1**	
	Alternative Method 1: for the 4th and 5th marks		
	Rearrange and differentiate the circle equation or differentiate implicitly	**(M1)**	Replaces the second M1.
	$\frac{dy}{dx}=\frac{3-x}{y+1}$ or $\frac{dy}{dx}=\frac{3-x}{\left(25-(x-3)^2\right)^{\frac{1}{2}}}$	**(A1)**	Replaces the second A1.

Question	Answer	Marks	Guidance
8	**Alternative Method 2: for the last 5 marks**		
	$\widehat{ACP} = \widehat{MAP} = \tan^{-1}\frac{4}{3}$ or identifying similar triangles *PMA* and *AMC*	**(M1A1)**	*C* is the circle centre, *P* is intersection of the two tangents, *M* is intersection of *PC* and the *y*-axis.
	$\tan MAP = \frac{PM}{4}, \frac{4}{3} = \frac{PM}{4}, PM = \frac{16}{3}$ or use of similar triangles	**(M1A1)**	
	P is $\left(\frac{-16}{3}, -1\right)$	**(A1)**	
	Alternative Method 3: for the last 5 marks		
	Pythagoras on triangle *PAC*, $PC^2 = PA^2 + AC^2$,	**(M1)**	Identifies the required 3 sides and sets up formula.
	$PC^2 = (PM+3)^2$, $PA^2 = PM^2 + 4^2$, $AC = \text{radius} = 5$	**(A1)**	Finds each side with two in terms of *PM* OE.
	$(PM+3)^2 = PM^2 + 4^2 + 5^2$ leads to $6PM = 32$, $PM = \frac{16}{3}$	**(M1A1)**	Sets up and solves equation.
	P is $\left(\frac{-16}{3}, -1\right)$	**(A1)**	
		8	

Question	Answer	Marks	Guidance
9(a)	Differentiate to obtain form $kx^2(2x^3+10)^{-\frac{1}{2}}$	**M1**	OE
	$3x^2(2x^3+10)^{-\frac{1}{2}}$	**A1**	Or unsimplified equivalent.
	Substitute $x=3$ in first derivative and evaluate to find gradient	***M1**	Expect $\frac{27}{8}$. Allow if first derivative of forms $k(2x^3+10)^{\frac{-1}{2}}$, $kx(2x^3+10)^{\frac{-1}{2}}$ or $kx^2(2x^3+10)^{\frac{-1}{2}}$.
	Attempt equation of tangent at $(3,8)$ with numerical gradient	**DM1**	Use of gradient of the normal is DM0.
	$[\pm](27x-8y-17)=0$ or integer multiples	**A1**	
		5	
9(b)	State or imply volume is $\pi\int(2x^3+10)\,dx$	**B1**	Implied if π appears only at the end. Do not allow an unsimplified: $\pi\int\left(\left(2x^3+10\right)^{1/2}\right)^2$.
	Integrate to obtain $k_1x^4+k_2x$ and evaluate using limits 1 and 3	**M1**	Where $k_1k_2\neq 0$.
	60π	**A1**	OE Allow from a correct integral and sight of limits. Allow numerical answers in the range 188-189.
		3	

Question	Answer	Marks	Guidance
10(a)	Substitute to obtain equation $9r^4+14r^2-8=0$	**B1**	OE
	Attempt solution of quadratic equation in r^2 to obtain at least one value of r or r^2	**M1**	Expect $(9r^2-4)(r^2+2)$.
	$r=\frac{2}{3}$ only	**A1**	**SC B1** answer without working.
		3	
10(b)	Substitute $a=2$ and *their* r in correct formula and attempt to evaluate	**M1**	Expect $\dfrac{2\left(1-\left(\frac{2}{3}\right)^{20}\right)}{\left(1-\frac{2}{3}\right)}$ or $\dfrac{2\left(\left(\frac{2}{3}\right)^{20}-1\right)}{\left(\frac{2}{3}-1\right)}$.
	5.998	**A1**	AWRT and no other value.
		2	
10(c)	Identify $a_2=\dfrac{4}{3}$ and common ratio as $\dfrac{8}{27}$.	**B1 FT**	Following *their r* provided $\lvert r\rvert<1$. May be implied in the sum to infinity. Allow $\left(\dfrac{2}{3}\right)^3$.
	Substitute *their* new a and r in correct formula for sum to infinity and evaluate	**M1**	$\lvert r\rvert<1$ otherwise M0.
	$\dfrac{36}{19}$	**A1**	OE Accept 1.89 or better from 1.894736…..
		3	

Question	Answer	Marks	Guidance
11(a)	Express $f(x)$ as: $a-(x-3)^2$ or $a-(3-x)^2$ where $a=\pm19$ or ±1	**M1**	OE If the form $-f(x)=(x^2-6x-10)$ is used the form must be returned to $f(x)=\ldots$ Completed square form must give $-x^2$. Answers must come from completion of the square (not calculus or graphs).
	$19-(3-x)^2$ or $19-(x-3)^2$	**A1**	OE
	$f(x)\leqslant19$ or $y\leqslant19$ with $\leqslant$, not $<$ or $-\infty<f(x)\leqslant19$ or $-\infty\leqslant f(x)\leqslant19$ or $(-\infty, 19]$ or $[-\infty, 19]$	**A1 FT**	Using *their* constant following the award of M1. **SC B1** answer only or answer from a method not involving completion of the square.
		3	

Question	Answer	Marks	Guidance
11(b)	$g^{-1}(x) = \frac{1}{4}(x - k)$	**B1**	
	$g^{-1}f(x) = \frac{1}{4}\left(10 + 6x - x^2 - k\right) = 4x + k$	**M1**	OE May use *their* completed square form for f(*x*).
	Simplify the quadratic equation obtained from $g^{-1}f(x) = g(x)$ provided *k* is present and apply $b^2 - 4ac = 0$ to this quadratic equation	***M1**	Expect $x^2 + 10x - 10 + 5k = 0$.
	Obtain $100 - 4(5k - 10) = 0$ and hence $k = 7$	**A1**	
	Use *their* *k* to form and solve a quadratic in x	**DM1**	Allow if *their* quadratic has two solutions.
	$(-5, -13)$ only	**A1**	**SC B1** if no method seen.
	Alternative Method for first 4 marks		
	State $f(x) = gg(x)$	**(B1)**	
	$gg(x) = 16x + 5k$	**(M1)**	
	Apply $b^2 - 4ac = 0$ to quadratic equation obtained from $f(x) = gg(x)$	**(*M1)**	Provided *k* is present.
	$100 - 4(5k - 10) = 0$ and hence $k = 7$	**(A1)**	
		6	